CONCRETE AND CONSTRUCTIONAL ENGINEERING

INCLUDING PRESTRESSED CONCRETE

UNIVERSITY OF MICHIGAN
JUN 30 1958
ENGINEERING LIBRARY

JUNE 1958 **VOL. LIII. NO. 6**

FIFTY-THIRD YEAR OF PUBLICATION

PRICE 2s. ANNUAL SUBSCRIPTION 24s. POST FREE. $3.90 in Canada and U.S.A.

LEADING CONTENTS

	PAGE
Simplicity and Economy	223
Beams and Slabs Designed by the Load-factor Method *By Chas. E. Reynolds*	225
Competition for the Design of a Factory	234
Pile Calculations by the Wave Equation *By Edward A. Smith*	239
Book Reviews	242
A Prestressed Bridge at Nottingham	244
Extending Precast Prestressed Piles	247

No. 607 ISSUED MONTHLY *Registered for Canadian Magazine Post*

BOOKS ON CONCRETE For Catalogue of "Concrete Series" books on concrete and allied subjects, send a postcard to:

CONCRETE PUBLICATIONS LTD., 14 DARTMOUTH ST., LONDON, S.W.1.

Ever used the Blue Circle Advisory Service?

THE Technical Department of the Blue Circle Group of Companies gives free practical advice on all matters concerning cement. Would you like a trained demonstrator to call and advise you or show how best to use Blue Circle decorative products? This can be arranged—in any part of the country. The Blue Circle Group is the largest cement organization in the world. Make use of its many services, profit from its great resources. Whenever you've got a cement problem, get in touch with us.

BLUE CIRCLE PRODUCTS Include

BLUE CIRCLE PORTLAND CEMENT · FERROCRETE · '417' CEMENT · SULFACRETE HYDROCRETE · SNOWCRETE · COLORCRETE · HYDRALIME · SNOWCEM, ETC.

A BLUE CIRCLE CMC PRODUCT

THE CEMENT MARKETING COMPANY LIMITED,
PORTLAND HOUSE, TOTHILL STREET, LONDON, S.W.1. TELEPHONE: ABBEY 3456.
G. & T. EARLE LIMITED HULL. TELEPHONE: HULL 16121.
THE SOUTH WALES PORTLAND CEMENT & LIME CO. LTD., PENARTH, GLAM. TELEPHONE: PENARTH 5736 1/4.

CONCRETE AND CONSTRUCTIONAL ENGINEERING

INCLUDING PRESTRESSED CONCRETE

Volume LIII, No. 6. LONDON, JUNE, 1958.

EDITORIAL NOTES

Simplicity and Economy.

A LARGE part of the training of young engineers is concerned with the efficient use of materials, and comparatively little with the cost of engineering works. It is not surprising, therefore, that the words efficiency and economy are sometimes considered to be synonymous; otherwise such misleading terms as "economic depth" and "economic ratio" would long since have been discarded. Most designers are brought to realise at an early stage of their careers that the simplest designs are generally the cheapest, even if they require more material than is really necessary. The provision of long lengths of vertical reinforcement in order to reduce the extra material at splices, or of complicated sections in order to save concrete, may call for less materials but is generally more expensive than the more wasteful alternatives. There can be few engineers who have never made such minor errors, and learned from them the value of simple design and detailing; it is therefore surprising that the extent of the saving in cost which is often made possible by simple planning is not so widely realised. Any requirement can generally be fulfilled in several different ways, and the cost of the resulting structure will depend to a very large degree upon the method selected. The adoption of an unnecessarily complicated scheme can result only in an unnecessarily expensive structure, yet the importance of simplicity at this stage of the design is frequently under-estimated.

In this number we illustrate the first premiated design in a competition for the design of a reinforced concrete factory. In the course of his report the assessor states that the impression given by the various structural schemes submitted is that most of the competitors created unnecessary difficulties by making their schemes unduly complicated. The assessor states that no system of construction lasts very long unless it is very simple, he expresses the view that the general standard of structural solutions submitted is disappointing, and urges designers to concentrate on making structures simple rather than spectacular. It is depressing that such comments should have been found to be necessary. It is to be expected that designs prepared as entries for competitions should tend towards the unusual, but it does not necessarily follow that such designs must be complex. If the competitors can only produce schemes which are either simple or spectacular, but not both, a simple scheme is to be preferred.

The simplest reinforced concrete roof for long spans is some form of shell structure ; yet surprise is expressed by the assessor that very few of the ninety-six entries submitted included proposals for shell roofs. The comment is made that while a shell roof may not be cheaper than a well-designed precast framework with light-weight sheeting, it would certainly be cheaper than many of the schemes submitted including precast and other forms of solid roofing. For the fairly large spans implied by the conditions of the competition this seems to be so self-evident that it is difficult to understand why so few shell roofs were proposed, and the assessor's suggestion that they may now be " out of fashion " is illuminating. If this is the case, then many clients may be called upon to spend more money on their buildings than they need to do because architectural and engineering " fashion " denies them a simpler, cheaper, and more efficient alternative. It is disturbing that so many of the schemes submitted should be based on such a premise.

A thorough acquaintance with the cost of engineering works discourages the production of complicated designs. It is stated in the report that variations in the estimates, due either to optimism or lack of consideration, were so pronounced as to constitute the main features of the various schemes submitted. There was a difference of more than one hundred per cent. between the lowest and highest estimates submitted, and it is stated that, although many of the structural schemes submitted would be extremely costly, this fact was not reflected in the estimates which accompanied the schemes. The estimates of cost per square foot of the manufacturing and storage areas of the three winning designs are given in the report as 50*s*., 51*s*. 7*d*., and 41*s*. 9*d*. It is possible to erect a structure which would serve the same purpose for a much lower cost ; such a structure would meet the requirement of the competition that the building must be economical, but it would be unlikely to comply with the spirit of the competition which called for " imaginative " designs that would " add to the amenities of an industrial area ". It seems that many of the competitors attached too much importance to the word imaginative, concentrated on constructional gymnastics and novelty, and forgot the advice of Francis Bacon to leave enchanted palaces to the poets who build them at small cost.

There can be few prospective owners of factories who would willingly pay so much more in order to improve the architectural quality of their buildings. Many of the light and economical factories which have recently been erected are by no means unpleasing, and many owners do not begrudge the extra cost of such structures ; however, the employers of architects and engineers are entitled to expect that their money should be used to the best advantage. The sanctioning of unnecessarily complicated structures when the architects' or engineers' estimates of cost are unrealistically low, and without getting an actual tender from a contractor, is no doubt infrequent and will remain so ; the report serves an excellent purpose in stressing the importance of simple designs and reliable, or even pessimistic, estimates of cost. Many of the designs would not detract from such amenities as an industrial area may possess, but according to the assessors few of them could be considered economical.

Beams and Slabs Designed by the Load-factor Method.*

In Accordance with the B.S. Code of 1957.

By CHAS. E. REYNOLDS, B.Sc. (Eng.), A.M.I.C.E.

Comparison of Load-factor and Modular-ratio Methods of Design.

Designs of beams by the load-factor method are compared in the following with corresponding designs by the modular-ratio method, which is based on the elastic behaviour of beams at working load.

RECTANGULAR BEAMS AND SOLID SLABS WITH TENSILE REINFORCEMENT ONLY.—The moment of compressive resistance of a rectangular beam or solid slab reinforced in tension only can be expressed as $Q_c.bd_1^2$. In the load-factor method Q_c is always $0{\cdot}25f_{cb}$ irrespective of the tensile stress in the reinforcement. In the modular-ratio method there is a complex relation between Q_c and the permissible stress f_{cb} in the concrete and the permissible or actual tensile stress f_{st} in the reinforcement. Values of Q_c calculated by the alternative methods are given in *Table* VII for mild steel bars and high-yield-stress bars and for ordinary, lower and higher grades of concrete. Comparison of the tabulated values of Q_c shows that the load-factor method allows a greater value for the

TABLE VII.

Reinforcement			Mild steel			High-yield-stress steel		
f_{st} (lb. per square inch)			20,000			30,000		
f_{st} (lb. per square inch)			18,000			23,000		
f_{cb} (lb. per square inch)			750	1000	1250	750	1000	1250
With tensile reinforcement only	Load-factor method	Q_c	187½	250	312½	187½	250	312½
		r_L	0·0125	0·0167	0·0208	0·0083	0·0111	0·0139
	Modular-ratio method	Q_c	119½	184	254	93	148	210
		r_L	0·0067	0·0107	0·0152	0·0034	0·0056	0·0080
With maximum amount of compressive reinforcement	Load-factor method ($A_{sc} = 0{\cdot}04bd$)	Q_c	900 av.	960 av.	1025 av.	1100 av.	1160 av.	1220 av.
		r_1 (A_{st})	0·052 av.	0·056 av.	0·061 av.	0·041 av.	0·044 av.	0·047 av.
		r_2 (A_{sc})	0·044 av.	0·044 av.	0·044 av.	0·046 av.	0·046 av.	0·046 av.
		$r_1 + r_2$	0·096 av.	0·100 av.	0·105 av.	0·087 av.	0·090 av.	0·093 av.
	Modular-ratio method ($A_{sc} = A_{st}$) $d_2 = 0{\cdot}1d_1$	Q_c	193	406	758	119	227	374
		$r_1 = r_2$ (A_{st})	0·011	0·023	{0·046 / 0·044}	0·0044	0·0082	0·0141
		$r_1 + r_2$	0·022	0·046	0·090	0·009	0·016	0·028

compressive resistance of the concrete than does the modular-ratio method, the difference being up to 100 per cent. for lower-grade concrete reinforced with high-yield-stress bars or wire, and about 40 per cent. for ordinary-grade concrete

* Concluded from May number.

reinforced with mild steel bars. The greater the strength of the concrete, as expressed by the permissible stress p_{cb}, the smaller the percentage difference; but the greater the tensile stress the greater the percentage difference. The area of tensile reinforcement required to provide a moment of tensile resistance equal to the moment of compressive resistance can be expressed by $r_L.bd_1$; values of r_L corresponding to the tabulated values of Q_c are also given in *Table* VII. This tensile reinforcement is greater for the load-factor method than for the modular-ratio method and varies directly with Q_c; direct comparison of the values of r_L does not, however, present a fair picture unless they are modified to suit the different moments of resistance or effective depths.

At first sight these differences in Q_c, and to a less extent in r_L, are serious divergences between two accepted methods of calculation, the practical implication of which can be examined by determining in general terms the sizes and reinforcement of rectangular beams and solid slabs required to resist a specified bending moment. Consider a beam or slab of ordinary-grade concrete (p_{cb} = 1000 lb. per square inch) with mild-steel tensile reinforcement only (p_{st} = 20,000 lb. per square inch) which is required to resist a bending moment of M' in.-lb. per inch of width. By the load-factor method

$$d_1 = \sqrt{\frac{M'}{250}} = 0{\cdot}0633\sqrt{M'} \text{ in.}$$

$$A_{st} = 0{\cdot}0167d_1 = 0{\cdot}00106\sqrt{M'} \text{ sq. in. per inch width.}$$

By the modular-ratio method

$$d_1 = \sqrt{\frac{M'}{184}} = 0{\cdot}074\sqrt{M'} \text{ in.}$$

$$A_{st} = 0{\cdot}0107d_1 = 0{\cdot}00079\sqrt{M'} \text{ sq. in. per inch width.}$$

By the load-factor method there is, therefore, a saving of concrete of $0{\cdot}13\sqrt{M'}$ cu. in. per foot length of beam or slab for each 1 in. of width, and a small saving in shuttering in the case of a beam; but there is an increase in net weight of reinforcement of $3{\cdot}4 \times 0{\cdot}00027\sqrt{M'} = 0{\cdot}00092\sqrt{M'}$ lb. per foot. At current prices of 1 : 2 : 4 concrete and mild steel bars the beam or slab designed by the load-factor method costs more than the deeper beam or thicker slab with less reinforcement given by the modular-ratio method.

Consider now two beams or slabs of the same effective depth as that required by the modular-ratio method, that is $0{\cdot}074\sqrt{M'}$ in. Since the quantities of concrete and shuttering in this case are identical, the only variation in cost is due to different amounts of tensile reinforcement. The amount of mild steel reinforcement required by the load-factor method is calculated from the appropriate design formula in series B in *Table* I, that is, A_{st} is $\frac{bd_1}{60}F$ where $F = 2\left(1 - \sqrt{1 - \frac{M'}{333bd_1^2}}\right)$; substituting $b = 1$ in. and $d_1 = 0{\cdot}074\sqrt{M'}$, then $A_{st} = 0{\cdot}00081\sqrt{M'}$ sq. in. per inch width compared with $0{\cdot}00079\sqrt{M'}$ sq. in. required by the modular-ratio method; the advantage is slightly in favour of the modular-ratio method, but the difference may be insignificant in practice.

RECTANGULAR BEAMS WITH COMPRESSIVE REINFORCEMENT.—The values of the moment-of-resistance factors Q_c for a rectangular beam with compressive reinforcement, and the corresponding coefficients of tensile reinforcement $\left(r_1 = \frac{A_{st}}{bd_1}\right)$ and compressive reinforcement $\left(r_2 = \frac{A_{sc}}{bd_1}\right)$, are given in *Table* VII for the same range of stresses as for a rectangular beam with tensile reinforcement only and for the two methods of calculation. The factors for the load-factor method are derived from *Tables* III and IV, and therefore the amount of compressive reinforcement is the maximum permissible, being 4 per cent. of the gross cross-sectional area of the beam. The limiting amount of compressive reinforcement for the factors for the modular-ratio method is that required for $A_{st} = A_{sc}$ when the compressive stress in the concrete is taken into account and does not exceed the permissible stresses stated in the table. For the modular-ratio method it is assumed that $d_2 = 0{\cdot}1d$. Except in the case of $p_{cb} = 1250$ lb. and $p_{st} = 20{,}000$ lb. per square inch, the amount of compressive reinforcement required by the modular-ratio method is much less than 4 per cent., and this fact, combined with the greater compressive resistance that is attributed to the concrete and the compressive reinforcement by the load-factor method, gives considerably greater moments of resistance (and quantities of reinforcement) when the load-factor method is used than when the modular-ratio method is used.

Comparative factors, similar to those previously derived for rectangular beams with tensile reinforcement only, for the common case of $p_{cb} = 1000$ lb. per square inch and mild steel bars are as follows.

For the smallest beam by the load-factor method,

$$d_1 = \sqrt{\frac{M'}{960}} = 0{\cdot}0323\sqrt{M'} \text{ in.}$$

$$A_{st} + A_{sc} = 0{\cdot}100d_1 = 0{\cdot}00323\sqrt{M'} \text{ sq. in. per inch width.}$$

For the smallest beam by the modular-ratio method,

$$d_1 = \sqrt{\frac{M'}{406}} = 0{\cdot}050\sqrt{M'} \text{ in.}$$

$$A_{st} + A_{sc} = 0{\cdot}046d_1 = 0{\cdot}0023\sqrt{M'} \text{ sq. in. per inch width.}$$

The saving in concrete obtained by the load-factor method is $0{\cdot}212\sqrt{M'}$ cu. in. per foot length of beam for each 1 in. of width, but the net weight of reinforcement is increased by $0{\cdot}0032\sqrt{M'}$ lb. per foot, the cost of which is more than the saving in the cost of concrete and shuttering.

Comparing two beams of identical size, both having an effective depth equal to that required for the smallest beam by the modular-ratio method, the area of compressive reinforcement required for a beam designed by the load-factor method is calculated by substituting $b = 1$ in., $d_2 = 0{\cdot}1d_1$, and $d_1 = 0{\cdot}050\sqrt{M'}$ in. in the appropriate design formula in series B in *Table* I, from which

$$A_{sc} = \frac{\frac{M'}{d_1^2} - 250}{18{,}000\left(1 - \frac{d_2}{d_1}\right)} bd_1 = 0{\cdot}00046\sqrt{M'} \text{ sq. in. per inch width.}$$

Similarly, the area of tensile reinforcement is calculated from

$$A_{st} = \frac{bd_1}{60} + 0{\cdot}9A_{sc} = 0{\cdot}0012\sqrt{M'} \text{ sq. in. per inch of width.}$$

The total reinforcement $A_{st} + A_{sc} = 0{\cdot}0017\sqrt{M'}$ sq. in. per inch of width compared with $0{\cdot}0023\sqrt{M'}$ sq. in. required by the modular-ratio method; design by the modular-ratio method is therefore the less economical if mild steel bars are used; but, again, consideration should be given to practical designs as in the examples which follow.

If high-yield-stress bars are used it is seen from *Table* VII that the values of Q_c for the load-factor method are greater than for mild steel bars, whereas the values of Q_c for the modular-ratio method are less for high-yield-stress bars than for mild steel bars. Moreover, the values of Q_c for high-yield-stress bars for beams with compressive reinforcement (modular-ratio method) are less than the values for the load-factor method without compressive reinforcement. It is therefore obvious that for beams of equal size with high-yield-stress bars, and a depth equal to the smallest required by the modular-ratio method, the load-factor method requires no compressive reinforcement, and, since the tensile reinforcement is less than the total reinforcement required by the modular-ratio method, the load-factor method again gives the more economical design.

FLANGED BEAMS.—In an ordinary tee-beam, in which the flange provides more compressive resistance than is required to resist the applied bending moment, the moment of resistance depends on the amount of tensile reinforcement. The load-factor formulæ for this condition (*Table* VI) are identical with those for the modular-ratio method, and there is therefore no difference in designs based on the two methods.

The moment of resistance of a tee-beam, ell-beam, or I-beam in which the compressive stress in the concrete is equal to, or nearly equal to, the permissible stress, is considerably greater if it is calculated by the load-factor method than if it is calculated by the modular-ratio method, and the amount of tensile reinforcement required to balance this compressive resistance is likewise greater. In general, the remarks on the relative economy of rectangular beams with and without compressive reinforcement designed by the two methods apply also to flanged beams if the compressive stress is equal to, or nearly equal to, the permissible stress.

Examples of Continuous Rectangular Beams and Solid Slabs.

The meaning of the foregoing differences, in terms of sizes and reinforcement, is best seen by comparing complete designs, because the foregoing comparisons apply to one section only of a rectangular beam or solid slab. In practice such comparisons would therefore apply only to simply-supported members or to continuous members if the bending moments at the supports are the same as that at midspan. The more common case of unequal bending moments at supports and midspan are considered in the following examples.

RECTANGULAR BEAM.

Calculation Sheets Nos. 1 and 2 give computations for six comparative designs of an interior span of a rectangular beam continuous over a number of spans

of 20 ft. and carrying a floor constructed of precast slabs. In all the designs the beam is 12 in. wide, as this width is required to provide a bearing for the slabs; the ratio of span to breadth is 20 and therefore the beam is not a slender beam (see *Table* 14 of the Code) and the maximum permissible compressive stress p_{cb}, in this case 1000 lb. per square inch, can be adopted. The depth of the beam in each case is determined by the resistance required at midspan, the design at this section being an example of a rectangular beam with tensile reinforcement only. Each beam is of uniform depth throughout the span, as this is most economical in shuttering; therefore, since the bending moment is greater at the supports than at midspan, the design of the section at the supports is an example of a rectangular beam with compressive reinforcement. The stiffness is satisfactory since the ratio of span to overall depth is in all cases less than 25 (see *Table* 13 of the Code).

The calculations show that the shearing stresses are satisfactory. Reinforcement is required near the supports in designs (b) and (e) to resist the shearing force; one or more of the bars provided for tensile reinforcement are available for bending up into the top of the beam. Such bars are used in all the designs to assist in providing the tensile resistance required over the supports; not all the bars over the supports are fully effective for the latter purpose and those that are bent down immediately at the edge of the support, as in designs (b) and (e), are not considered in computing the tensile resistance. The bars remaining in the bottom of the beam at the supports overlap with those from adjacent spans to provide the necessary compressive reinforcement. The arrangements of the bars are shown in the diagrams on the calculation sheets. If additional resistance to shearing forces were required, this would be provided by mild steel binders; if no such extra resistance were required, nominal binders of mild steel would be provided.

The designs (a), (b) and (c) on Calculation Sheets Nos. 1 and 2 are for mild steel bars. Design (a) is in accordance with the modular-ratio method with the concrete stressed to 1000 lb. per square inch. Design (b) is similar but in accordance with the load-factor method and is based on *Table* III. Design (c) is in accordance with the load-factor method but with the same depth of beam as is required for the modular-ratio design (a); the formulæ in design (c) are from *Table* I and the factors are from *Table* II.

The three designs (d), (e), and (f) on Calculation Sheet No. 2 correspond to (a), (b), and (c) respectively but are for high-yield-stress bars; therefore design (e) is based on *Table* IV and design (f) on *Tables* I and II.

The relative economy of the six designs can be determined by inspecting the calculations and diagrams. Although a fair comparison can be made only by considering complete designs such as these, it must be remembered that a beam is usually of practical depth and is therefore likely to be deeper than the calculated minimum. Similarly the area of reinforcement may exceed the area required because the area of a whole number of bars of a size commercially obtainable is likely to be greater than the calculated minimum area. These excesses may favour one design slightly more than another.

The principal conclusions arising from an examination of the six designs are

(i).—The modular-ratio method results in a more economical design than

CALCULATION SHEET NO. 1.

RECTANGULAR BEAMS SUPPORTING PRECAST FLOOR SLABS.

INTERIOR SPANS
$l = 20$ ft.
Beams at 8 ft. centres.
$b = 12"$

Dead load: Precast slabs, topping, etc. = 45 lb. per sq. ft.
Partitions = 20
Ceiling finish, etc. = 5
70×8 ft. = 560 lb. per ft.
Beam, etc., say, = 240
Total dead load = 800
Live load: 80 lb. per sq. ft. $\times$ 8 ft. = 640

For all Designs.

Bending Moments.

At midspan. $M = \frac{800 \times 20^2 \times 12}{24} + \frac{640 \times 20^2 \times 12}{12} =$ 416,000 in.-lb. (34,700 in.-lb. per inch width)

At supports. $M = \frac{800 \times 20^2 \times 12}{12} + \frac{640 \times 20^2 \times 12}{9} =$ 661,000 in.-lb. (55,100 in.-lb. per inch width)

Shearing Force $= (800 + 640) \times 10$ ft. $=$ 14,400 lb.

CONCRETE.
REINFORCEMENT.

Ordinary grade 1:2:4. $f_{cb} = 1000$ lb. per sq. in.
Mild steel bars. $f_{st} = 20{,}000$ lb./sq. in. $f_{sc} = 18{,}000$ lb./sq. in.

For Designs (a), (b) & (c).

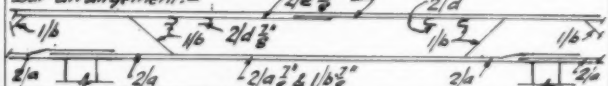
DESIGN (a) MODULAR-RATIO METHOD

$m = 15$
$Q_c = 184$
$a_1 = 0.86$
$n_1 = 0.428$
$m - 1 = 14$

Midspan: $d_1' = \sqrt{\frac{416{,}000}{12 \times 184}} = 13.7"$. $d = 16"$; $d_1 = 14.5"$
$l_a = 0.86 \times 14.5 = 12.5'$
$A_{st} = \frac{416{,}000}{20{,}000 \times 12.5} = 1.67$ sq. in.

Support: $d = 16"$; $d_1 = 14.5"$; $d_2 = 2"$; $l_a = 12.5'$; $\frac{1}{8} = 14.5 + 12.5$
$d_1 = 0.428 \times 14.5 = 6.2"$
M_r (concrete) $= 184 \times 12 \times 14.5^2 = 466{,}000$ in.-lb.
M_r (reinforcement) $= 661{,}000 - 465{,}000 = 196{,}000$ in.-lb.
$f_{sc} = \frac{1000 \times 14 (6.2 - 2)}{6.2} = 9{,}600$ lb./sq. in.
$A_{sc} = \frac{196{,}000}{9500 \times 12.5} = 1.65$ sq. in.
$A_{st} = \frac{466{,}000}{20{,}000 \times 12.5} + \frac{196{,}000}{20{,}000 \times 12.5} = 2.64$ sq. in.
$q = \frac{14{,}400}{12 \times 12.5} = 96$ lb./sq. in. Nominal binders.

Bar arrangement:—



16" × 12"
Midspan: 3-7/8" in bottom (1.80 sq. in.)

Over supports: 4-7/8" provided in bottom (2.41 sq. in.)
Provided in top: 4-7/8" / 2-5/8" (3.02 sq. in.) In one layer

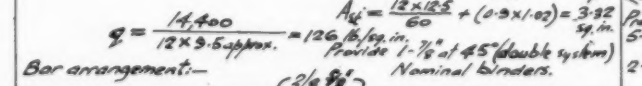
DESIGN (b) LOAD-FACTOR METHOD

$Q_c = 250$

Midspan: $M_r' = 34{,}700$ in.-lb./in. From table: $d = 14"$
$A_{st} =$ approx. $0.2 \times 12 = 2.4$ sq. in.

Support: $d = 14"$; $d_1 = 12.5"$; $d_2 = 2"$; $Q_r = \frac{661{,}000}{12 \times 12.5^2} = 353$.
From tabulated formulae (Series B) $A_{sc} = \frac{(353 - 250) 12 \times 12.5}{18{,}000 (1 - \frac{2}{12.5})} = 1.02$ sq. in.
$A_{st} = \frac{12 \times 12.5}{60} + (0.9 \times 1.02) = 3.32$ sq. in.
$q = \frac{14{,}400}{12 \times 9.6 \text{ approx.}} = 126$ lb./sq. in. Provide 1-7/8" at 45° (double system) Nominal binders.

Bar arrangement:—



14" × 12"
4-7/8" in bottom (2.41 sq. in.) at midspan.

At support: 2-7/8" effective in bottom (1.20 sq. in.)
Provided in top: 5-7/8" effective / 2-5/8" (3.62 sq. in.)

(Continued on Calculation Sheet No. 2.)

CALCULATION SHEET NO. 2.

(Continued from Calculation Sheet No. 1.)

RECTANGULAR BEAMS SUPPORTING PRECAST FLOOR SLABS (cont.)

DESIGN(c) LOAD-FACTOR METHOD ($d = 16''$) $Q_c = 250$

Midspan: $d = 16''$; $d_1 = 14{\cdot}5''$; $Q_1 = \frac{416,000}{12 \times 14{\cdot}5^2} = 165 \ (< 250)$ $\therefore d_1 > d_1''$

From tables: $F = 0{\cdot}58$; $A_{st} = \frac{12 \times 14{\cdot}6}{60} \times 0{\cdot}58 = 1{\cdot}68$ sq. in.

Support: $d = 16''$; $d_1 = 14{\cdot}5''$; $d_2 = 2''$; $Q_1 = \frac{661,000}{12 \times 14{\cdot}5^2} = 262$

From tabulated formulæ (B):

$$A_{sc} = \frac{(262 - 250)}{18,000\left(1 - \frac{2}{14{\cdot}5}\right)}(12 \times 14{\cdot}5) = 0{\cdot}13 \text{ sq. in.}$$

$$A_{st} = \frac{12 \times 14{\cdot}5}{60} + (0{\cdot}9 \times 0{\cdot}13) = 3{\cdot}02 \text{ sq. in.}$$

$q <$ 100 lb. per sq. in. Nominal binders.

Arrangement of bars: As Design (a) but with minimum lap in bottom over supports.

$16'' \times 12''$

3-$1\frac{7}{16}''$ in bottom ($1{\cdot}80$)

2-$\frac{7}{8}''$ effective ($1{\cdot}20$ sq. in.) provided in bottom.

Provided in top:- 4-$\frac{7}{8}''$ & 2-$\frac{5}{8}''$ ($3{\cdot}02$ sq. in.)

CONCRETE. Ordinary grade 1:2:4. $f_{cb} = 1000$ lb. per sq. in.

REINFORCEMENT:- High-yield-stress bars: $f_{st} = 30,000$ lb./sq. in. $f_{sc} = 23,000$ lb./sq. in.

For Designs (d), (e) & (f).

DESIGN(d) Modular-Ratio Method $Q_c = 148$ $\eta = 0{\cdot}333$ $a_1 = 0{\cdot}89$ $m = 15$ $m - 1 = 14$

Midspan: $d_1' = \sqrt{\frac{416,000}{12 \times 148}} = 15{\cdot}3''$. $d = 18''$; $d_1 = 16{\cdot}63''$.

$l_a = 0{\cdot}89 \times 16{\cdot}63 = 14{\cdot}8''$

$$A_{st} = \frac{416,000}{30,000 \times 14{\cdot}8} = 0{\cdot}94 \text{ sq. in.}$$

Support: $d = 18''$; $d_1 = 16{\cdot}63''$; $l_a = 14{\cdot}8''$; $d_2 = 1\frac{3}{4}''$; $l_s = 16{\cdot}63 - 1{\cdot}75 = 14{\cdot}88''$

$d_n = 0{\cdot}333 \times 16{\cdot}63 = 5{\cdot}54''$

M_r (concrete) $= 148 \times 12 \times 16{\cdot}63^2 = 491,000$ in.-lb.

M_r (reinforcement) $< 661,000 - 491,000 = 170,000$ in.-lb.

$$p_{sc}' = \frac{1000 \times 14\left(5{\cdot}54 - 1{\cdot}75\right)}{5{\cdot}54} = 3560 \text{ lb./sq. in.}$$

$$A_{sc} = \frac{170,000}{3560 \times 14{\cdot}88} = 1{\cdot}20 \text{ sq. in.}$$

$$A_{st} = \frac{491,000}{30,000 \times 14{\cdot}8} + \frac{170,000}{30,000 \times 14{\cdot}88} = 1{\cdot}49 \text{ sq. in.}$$

$q <$ 100 lb./sq. in. Nominal binders.

Arrangement of bars: Similar to Design (a) but with l/d only.

$18'' \times 12''$

3-$\frac{3}{4}''$ in bottom. ($1{\cdot}33$ sq. in.)

4-$\frac{3}{4}''$ ($1{\cdot}77$ sq. in.) provided in bottom.

Provided in top: 3-$\frac{3}{4}''$ & 2-$\frac{1}{2}''$ ($1{\cdot}72$ sq. in.)

DESIGN(e) LOAD-FACTOR METHOD $Q_c = 250$

Midspan: $M_r'' < 34,700$ in.-lb./in. From table: $d = 14''$

Approx. $A_{st} = 12 \times 0{\cdot}140 = 1{\cdot}68$ sq. in.

Support: $d = 14''$; $d_1 = 12{\cdot}5''$; $d_2 = 2''$; $Q_1 = \frac{661,000}{12 \times 12{\cdot}5^2} = 363$.

From tabulated formulæ (C):-

$$A_{sc} = \frac{353 - 250}{23,000\left(1 - \frac{2}{12{\cdot}5}\right)}(12 \times 12{\cdot}5) = 0{\cdot}80 \text{ sq. in.}$$

$$A_{st} = \frac{12 \times 12{\cdot}5}{90} + (0{\cdot}77 \times 0{\cdot}80) = 2{\cdot}28 \text{ sq. in.}$$

$q >$ 100 lb./sq. in. Provide 1-$\frac{7}{8}''$ at 45° (double system). Nominal binders.

Arrangement of bars: Similar to Design (b) but with $3/d$ and excluding $1/c$.

$14'' \times 12''$

3-$\frac{7}{8}''$ in bottom.

2-$\frac{7}{8}''$ effective in bottom ($1{\cdot}20$ sq. in.)

Provided in top:- 4-$\frac{7}{8}''$ effective & 2-$\frac{5}{8}''$ ($2{\cdot}90$ sq. in.)

DESIGN(f) LOAD-FACTOR METHOD ($d = 18''$) $Q_c = 250$

Midspan: $d = 18''$; $d_1 = 16{\cdot}63''$; $Q_1 = \frac{416,000}{12 \times 16{\cdot}63^2} = 125 \ (< 250)$ $\therefore d_1 > d_1''$

From tables: $F = 0{\cdot}42$; $A_{st} = \frac{12 \times 16{\cdot}63}{90} \times 0{\cdot}42 = 0{\cdot}93$ sq. in.

Support: $d = 18''$; $d_1 = 16{\cdot}63''$; $d_2 = 1\frac{3}{4}''$; $Q_1 = \frac{661,000}{12 \times 16{\cdot}63^2} = 199 \ (< 250)$

$F = 0{\cdot}73$; $A_{st} = \frac{12 \times 16{\cdot}63}{90} \times 0{\cdot}73 = 1{\cdot}62$ sq. in.

Arrangement of bars: As Design (d) but with minimum lap.

$18'' \times 12''$

3-$\frac{3}{4}''$ in bottom ($1{\cdot}33$ sq. in.)

$\therefore A_{sc} = 0$

Provided in top: 3-$\frac{3}{4}''$ & 2-$\frac{1}{2}''$ ($1{\cdot}72$ sq. in.)

the load-factor method if the stress in the concrete in both designs is the maximum permissible; the reduction in the quantities of concrete and shuttering in the load-factor design is insufficient to offset the cost of the greater amount of reinforcement; compare designs (a) and (b) and designs (d) and (e).

(ii).—If the beam is designed by the load-factor method but is of the size required by the modular-ratio method, the amount of reinforcement is the same in both designs; compare designs (a) and (c) and designs (d) and (f). A slight advantage may lie with the design by the load-factor method because of the smaller amounts of compressive reinforcement required over the supports.

(iii).—Beams with high-yield-stress bars require a smaller weight of steel than equivalent beams with mild-steel bars; compare designs (a) and (d), (b) and (e), and (c) and (f); a small increase in the quantity of concrete is required by the modular-ratio method, but, provided that the prices of high-yield-stress bars are competitive, beams with these bars are more economical. Conclusions (i) and (ii) apply also to beams with high-yield-stress bars.

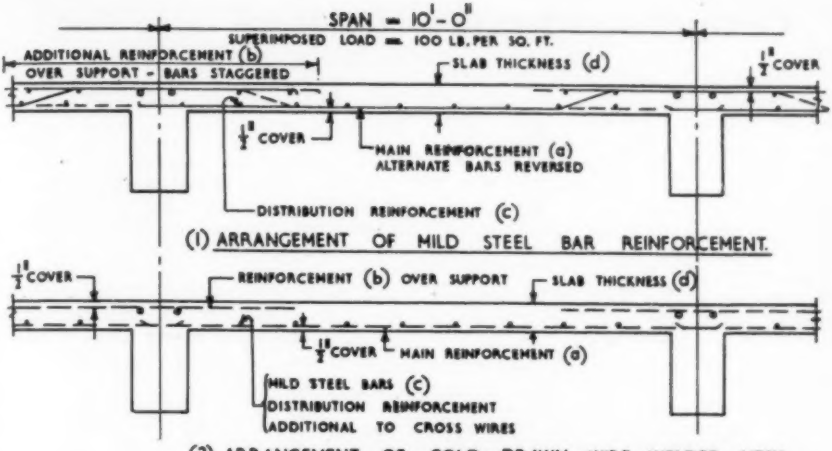
Designs (a) to (f) are for beams with ordinary-grade concrete. If similar designs are prepared for, at one extreme, lower-grade concrete reinforced with mild steel bars and, at the other, with higher-grade concrete with high-yield-stress bars, it will be seen that conclusions (i), (ii), and (iii) still apply. It is interesting to note that the weight of steel required in a beam of lower-grade concrete reinforced with mild steel bars is about the same as the amount required in a beam of higher-grade concrete reinforced with high-yield-stress bars so long as the basis of design is the same in each case. The former beam would be the larger, and the economic criterion is whether the unit costs of lower-grade concrete and mild steel bars are sufficiently low to offset the additional quantities of concrete and shuttering.

Solid Slab.

Consider a solid slab continuous over two or more equal spans. The bending moment is numerically greater at the supports than at midspan. Since it is not common to provide effective compressive reinforcement in a slab, and since it is more economical in shuttering if the slab is the same thickness throughout, the thickness provided should be the minimum required to resist the larger bending moment at the supports. At midspan, therefore, the thickness is greater than the minimum required to resist the smaller bending moment, and the amount of reinforcement is considerably reduced accordingly. Comparison of several designs for a slab spanning 10 ft., carrying a superimposed load of 100 lb. per square foot, reinforced with mild steel bars or cold-drawn wire fabric (with additional mild steel bars for distribution reinforcement), and designed by the modular-ratio or load-factor methods, are given in *Fig.* 1 from which the following conclusions can be drawn.

(i).—With mild steel bars, the more economical slab is that designed by the modular-ratio method; but there is no difference in the costs of the slabs whether designed by the modular-ratio method or load-factor method if the thickness is the minimum required by the modular-ratio method.

(ii).—With cold-drawn wire fabric as the principal reinforcement, the load-factor method results in a cheaper slab, which is also cheaper than the cheapest slab with mild steel bars. In the designs illustrated, the reinforcement over


SPAN = 10'-0"
SUPERIMPOSED LOAD = 100 LB. PER SQ. FT.
ADDITIONAL REINFORCEMENT (b) OVER SUPPORT - BARS STAGGERED
SLAB THICKNESS (d)
½" COVER
½" COVER
MAIN REINFORCEMENT (a) ALTERNATE BARS REVERSED
DISTRIBUTION REINFORCEMENT (c)
(1) ARRANGEMENT OF MILD STEEL BAR REINFORCEMENT.
½" COVER
REINFORCEMENT (b) OVER SUPPORT
SLAB THICKNESS (d)
½" COVER
MAIN REINFORCEMENT (a)
MILD STEEL BARS (c) DISTRIBUTION REINFORCEMENT ADDITIONAL TO CROSS WIRES
(2) ARRANGEMENT OF COLD-DRAWN WIRE WELDED MESH. (HIGH-TENSILE REINFORCEMENT)

TYPE OF REINFORCEMENT	METHOD OF DESIGN	SLAB (d)	REINFORCEMENT (a) MIDSPAN	(b) SUPPORT	(c) DISTRIBUTION	ECONOMIC RATIO
MILD STEEL BARS P_{st} = 20,000 LB./SQ. IN.	MODULAR RATIO	4"	½" AT 10"c/c	PLUS ½" AT 20"c/c	$\frac{3}{16}$" AT 12"c/c	1·00
		4½"	$\frac{3}{8}$" AT 6"c/c	PLUS $\frac{3}{8}$" AT 9"c/c	$\frac{3}{8}$" AT 15"c/c	1·13
	LOAD FACTOR	3½"	½" AT 8"c/c	PLUS ½" AT 16"c/c	$\frac{3}{16}$" AT 12"c/c	1·03
		4"	½" AT 10"c/c	PLUS ½" AT 20"c/c	$\frac{5}{16}$" AT 12"c/c	1·00
		4½"	$\frac{3}{8}$" AT 7"c/c	PLUS $\frac{5}{16}$" AT 7"c/c	$\frac{3}{8}$" AT 15"c/c	1·09
COLD-DRAWN WIRE WELDED MESH P_{st} = 30,000 LB./SQ. IN.	MODULAR RATIO	4"	INSUFFICIENT COMPRESSION			—
		4½"	B.S. REF. No. 109	B.S. REF. No. 107	$\frac{3}{8}$" AT 18"c/c	1·01
		5"	" " No. 110	" " No. 107	$\frac{3}{8}$" AT 15"c/c	1·08
	LOAD FACTOR	4"	" " No. 109	" " No. 106	$\frac{3}{16}$" AT 15"c/c	0·96
		4½"	" " No. 109	" " No. 107	$\frac{3}{8}$" AT 18"c/c	1·01
		5"	" " No. 110	" " No. 107	$\frac{3}{8}$" AT 15"c/c	1·08

Fig. 1.

the supports is provided by separate sheets of fabric; the economy would be even more pronounced if the bottom sheets were waved up over the supports.

The designs with the thickness of the slab greater than the minimum required at the supports (and with less reinforcement accordingly) show that the design procedure described in the foregoing, namely, basing the thickness on the minimum required at the supports, produces in general the cheapest slab.

Competition for the Design of a Factory.

FIRST PREMIATED DESIGN.

THE result of the competition sponsored by the Cement Marketing Co., Ltd., for the design of a reinforced concrete factory was given in our last number. The first prize of £1000 was awarded to Mr. J. Mendleson and Mr. J. H. C. Lamb (of Messrs. Joseph Mendleson & Partners), architects, in association with Mr. Z. L. J. Woloszczuk and Mr. R. B. Wilcox (of Messrs. Chamberlain & Partner), engineers, for the design shown in *Figs.* 1 to 3 and described in the following.

DESIGN.—A module of 3 ft. 6 in. would be used throughout. Most of the structural members would be precast and some would be prestressed. In order to obtain uniform illumination in the main production area the longitudinal axis of the factory would be in the north–south direction, the main roof glazing facing north and supplementary glazing facing south.

Long-span shallow prestressed beams at 3 ft. 6 in. centres would be used for the roofs of the canteen and entrance halls, with wood-wool slabs 2 in. thick spanning between them. The frame would be cast in place.

The main structural units of the factory roof would be triangular space-frames (*Figs.* 4 to 6) formed by two inclined Warren girders connected at the top by precast concrete planks and at the bottom by rectangular tie-beams at 6-ft. centres. The space-frames would form continuous lanterns at 28-ft. centres. Precast purlins would be supported on the bottom booms of the girders and would support wood-wool slabs. The largest triangular frame would weigh 7 cwt.

ERECTION.—The frames would be assembled below their final positions on sleepers at 6-ft. centres with gaps 1 in. wide between adjacent triangles, and arranged to provide a camber of 2 in. The apex of each triangle would be supported by a prop.

After the slabs forming the roof of the lantern had been erected, the gaps between them would be shuttered on the underside and the two triangular frames and the slabs between them connected by a joint

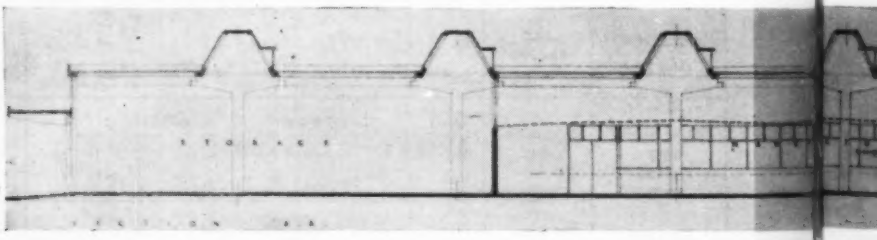


Fig. 3.—Lo dinal

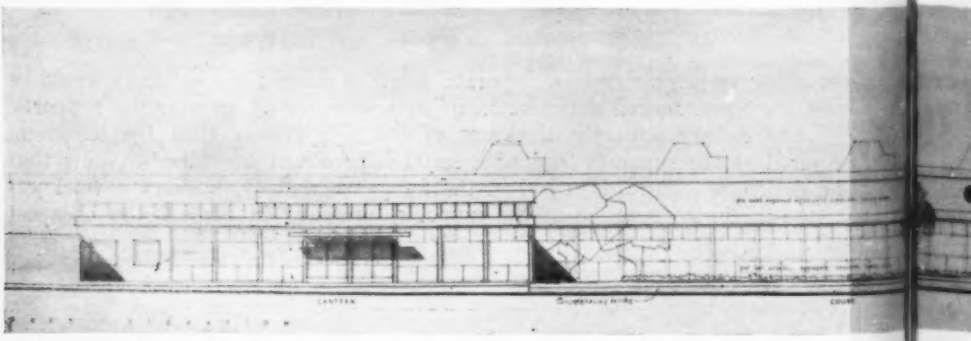


Fig. 3.—E levati

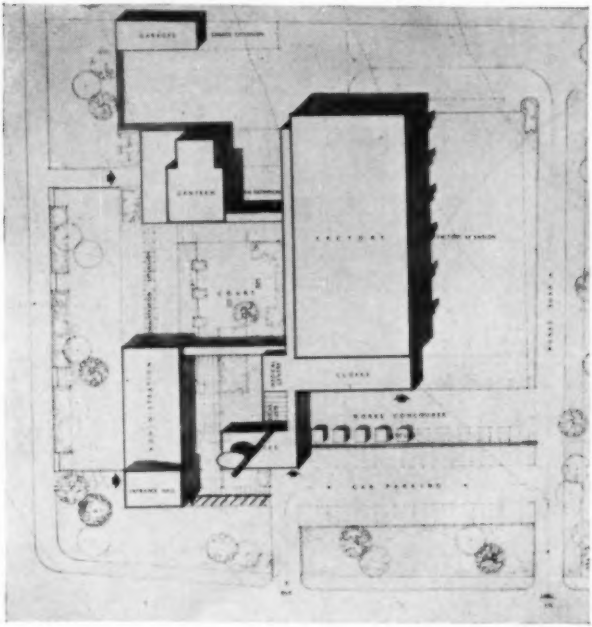
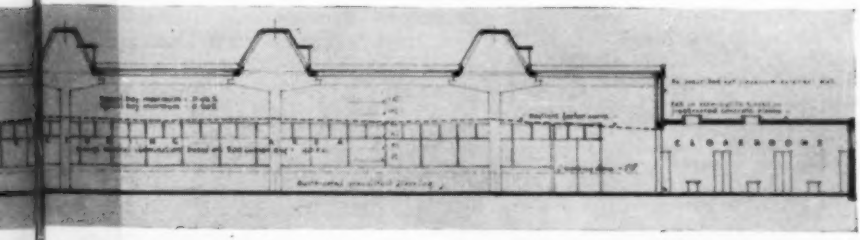
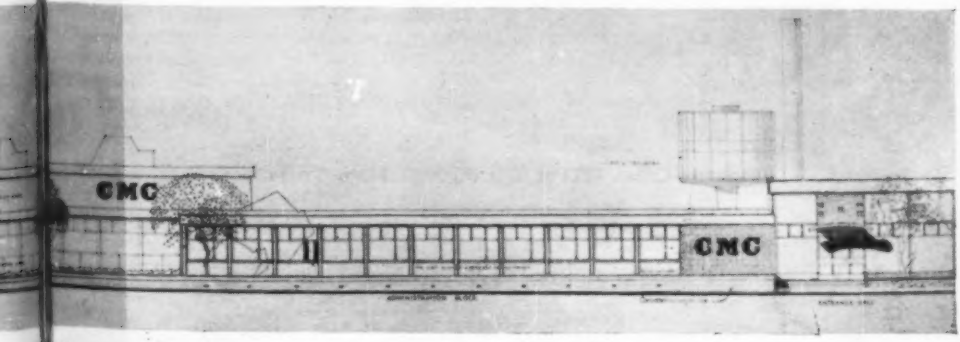


Fig. 1.—Site Plan.



-Longitudinal Section.



3.—Elevation.

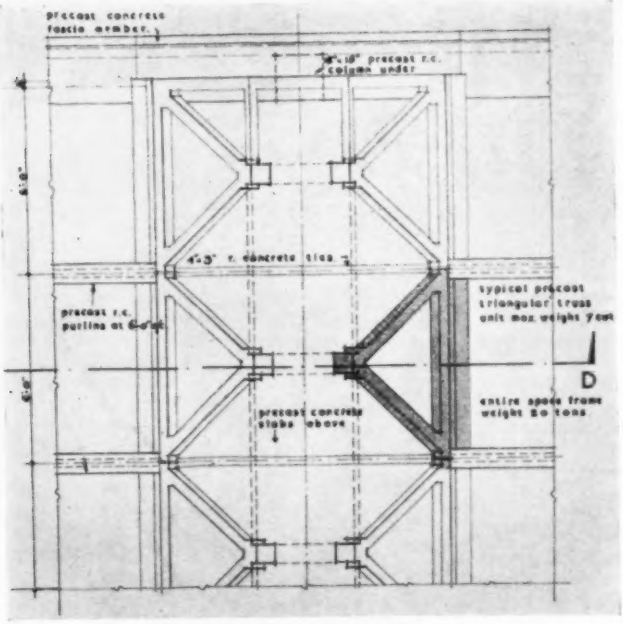

precast concrete fascia member
precast r.c. purlins at
4'-3" r. concrete ties
typical precast triangular truss unit max. weight 7 ton
D
entire space frame weight 80 tons
precast concrete slabs above

Fig. 4.—Plan of Main Truss.

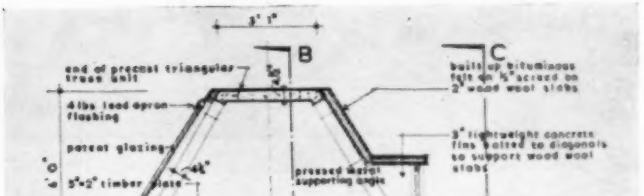

B
C
end of precast triangular truss unit
built up bituminous felt on ½" screed on 2" wood wool slabs
4 lbs. lead apron flashing
patent glazing
3" lightweight concrete fins bolted to diagonals to support wood wool slabs
3"×2" timber plate
pressed metal supporting angle
galvanised metal windows with r.c.h. opening lights
pressed metal cill
precast edge beam
bolted connection
dowel joint
10' 6"
dowel joint
bottom booms prestressed with 4 nos 0.276 in. diam. h.t. steel wires

Fig. 5.—Cross Section DD through Main Truss.

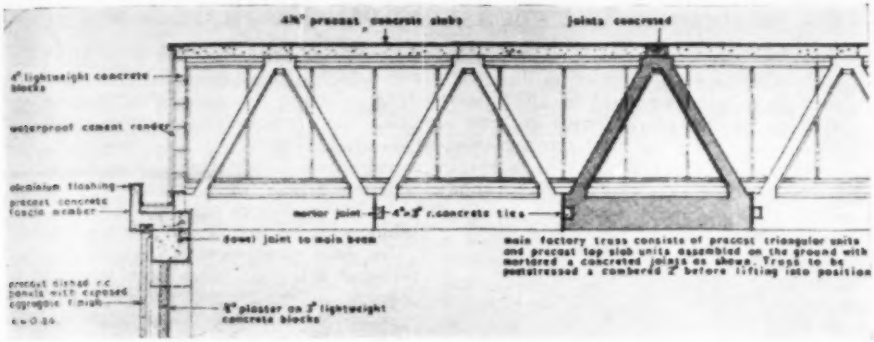

4½" precast concrete slabs
joints concreted
4" lightweight concrete blocks
waterproof cement render
aluminium flashing
precast concrete fascia member
mortar joint
4"×3" r.concrete ties
dowel joint to main beam
precast tinted r.c. panels with exposed aggregate finish
½" plaster on 3" lightweight concrete blocks
main factory truss consists of precast triangular units and precast top slab units assembled on the ground with mortared & concreted joints as shown. Trusses to be prestressed & cambered 2" before lifting into position

Fig. 6.—Section BB through Main Truss.

which would be cast in place. Prestressing cables would be passed through cavities formed in the bottom boom, bridging pieces being placed in the gaps between adjacent units, and the cavities filled with mortar. When the joints were strong enough the cables would be tensioned and the duct grouted. The complete frame, weighing 20 tons, would then be lifted on to T-shaped columns; these would be tightly wedged in pockets formed in the bases until the dowelled connections between the frame and the columns were made. The columns would be plumbed and concreted in their bases, and the dowel holes grouted.

The frames are designed to have pin-joints at the top of the columns, and all the wind forces would be resisted by the columns alone. In the longitudinal direction the columns would be connected by precast beams which would also form the last purlin supporting the roof covering and carry the precast gutters.

EXTERNAL FINISHES.—The exposed concrete frame would be left fair-faced. Precast panels would have exposed-aggregate surfaces, using coloured aggregate and coloured cement for the facing. The aggregate for the upper slabs would be from 1 in. to ¾ in. in size, and from ¾ in. to ¼ in. for the lower panels and the precast blocks. On some parts of the elevations profiled interlocking fair-face panels would be used. Coloured concrete facing bricks and panels of coloured cement would also be used.

COST.—The average cost of the manufacturing and storage areas is estimated to be 2*s.* 3*d.* per cubic foot, or 50*s.* per square foot. The average cost of the offices, canteen, lavatories, first-aid rooms, and boiler-house is estimated to be 4*s.* per cubic foot, or 74*s.* 2*d.* per square foot. The assessor (Mr. Edward D. Mills, F.R.I.B.A., assisted by Messrs. Ove Arup & Partners, and Mr. Leslie W. Clark, F.I.A.S.) states that this estimate is realistic, and that the authors have given more than the average care in its preparation.

INSULATION.—Generally the external walls would be of the cavity type, comprising precast concrete slabs and an inner leaf of lightweight concrete blocks. The finishes would not be continuous with the structure, and where possible panel walls would be separated from the structural frame and the floor.

[The assessor's report on this competition is referred to on p. 233.]

A Precast Shell Roof.

TWELVE precast concrete curved slabs with edge-beams, designed as "shells", each 24 ft. square and 1½ in. thick, were used to cover a market with an area of 96 ft. by 72 ft. in New Canaan, Connecticut, U.S.A. The shape of each slab is generated by a circular arc with a chord of 24 ft. and a rise of 3 ft. 6 in., the path of which is an identical circular arc. *Fig.* 1 shows the erection of one of the slabs, and details of the reinforcement are shown in *Fig.* 3.

Plywood shuttering, supported on curved timber ribs, was used (*Fig.* 2). Insulating blocks were placed on the shuttering before the concrete was placed. The slabs were erected (*Fig.* 1) by two cranes, using an equalising bar and a square lifting frame, and were waterproofed with three coats of acrylic plastic paint. The curved sides of the slabs were covered with translucent reinforced plastic to form roof lights. We are indebted to "Engineering News-Record" for the foregoing and for the illustrations.

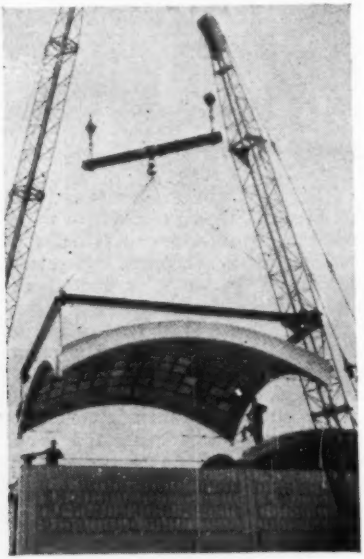


Fig. 1.—Erecting a Roof Slab.

Fig. 2.—Shutter for a Slab.

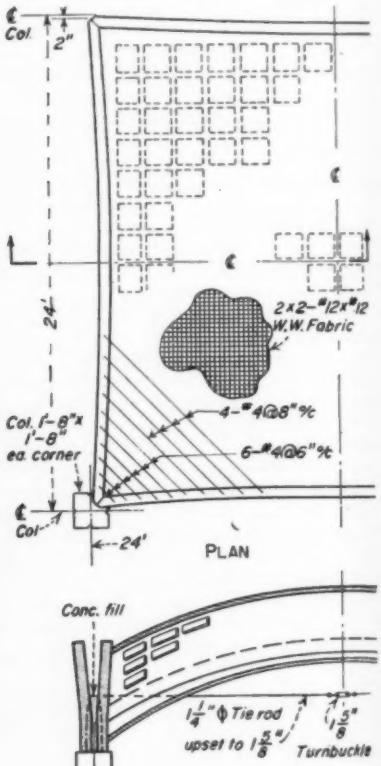


Fig. 3.—Details of Reinforcement.

Pile Calculations by the Wave Equation.

By EDWARD A. SMITH.

THE results of experiments in which precast concrete piles were driven, with piezo-electric quartz strain-gauges embedded in the concrete, were described in a paper published in 1938.[1] Gauges were located at the head, centre, and point of each pile, and the oscillograph records showed that the impact stresses progressed from the head of the pile to the point and back again in general accord with the laws of longitudinal transmission of stress in solids given by the wave equation

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}.$$

In Appendix I of the paper a wave-equation solution for pile-driving was presented. This solution was complicated, and included the simplifying assumption that the ground acted as a spring, whereas actually the ground has elastic-plastic characteristics. As a result, this mathematical solution was not considered practical, and pile-driving practice has continued to be based on formulæ which are either empirical or have a questionable mathematical basis.

With the increasing availability of electronic computers the wave equation can now be applied to pile-driving, taking into account all the variables which are known or can reasonably be assumed. The analysis can include all the variables used in Mr. Hiley's formula, such as the coefficient of restitution of the dolly or packing and the elastic-plastic action of the ground; friction and hysteresis can also be included, and the wave action is included automatically. The object of the calculation may be limited to determining the penetration of the point of the pile against a specified ultimate resistance to driving, or may be extended to give the compressive and tensile stresses occurring during driving. Such additional data as velocities, accelerations, and rates of compression are also obtainable if they are wanted.

Some analogue computers can solve the problem, but so far as the writer is aware no one has yet used one for this purpose; the degree of accuracy obtainable with such computers might be insufficient. On the other hand, the basic methods involved in making wave-equation calculations on digital computers and checking them for accuracy have been set out by the writer.[2], [3] The programming of an electronic digital computer in New York has been completed and a few pile-driving calculations have been made. All these calculations have included the coefficient of restitution of the dolly or packing, and the elastic-plastic resistance of the ground at the point of the pile. In some of the calculations elastic-plastic friction at the side has also been included. In all cases the quake of the ground (or elastic ground compression) was assumed to be 0·1 in., which is considered to be conservative.

Mr. Hiley's piling formula is probably the most rational of those in general use, and has terms that take account of all the major variables. However, the wave equation shows that some of the assumptions on which it is based are not always justified. For example, these assumptions make little difference for short piles or piles that are long but comparatively light, but when the formula is applied to piles that are long and heavy it leads to over-driving. This is objectionable in the case of precast piles because over-driving may shatter the concrete.

Consider the following example of a precast prestressed pile with post-tensioned steel, which is typical of one type of pile being driven in the U.S.A.

Length of pile, 150 ft.; outside diameter, 54 in.; inside diameter, 44 in.; weight per ft., 800 lb.; total weight, 120,000 lb.; cross section, 770 sq. in.; modulus of elasticity, about 5,000,000 lb. per square inch.

Details of the hammer are: weight of ram, 15,000 lb.; gravity fall, 39 in.; rated energy per blow, 48,750 ft.-lb.; assumed efficiency, 80 per cent.

If Mr. Hiley's formula is applied to such a pile and hammer, and if an ultimate driving resistance of 600,000 lb. is assumed, then the formula, with the usual constants, gives the result that the pile cannot be driven. However, such piles have been driven and test-loaded to

TABLE I.

Specified ultimate driving resistance (lb.)	Maximum compressive forces		Permanent penetration or set per blow (in.)	Blows per inch
	Near head (lb.)	Near point (lb.)		
600,000	702,000	875,000	0·199	5
900,000	702,000	1,177,000	0·080	12·5
1,200,000	702,000	1,364,000	0·0001	Refusal

800,000 lb. with negligible settlements and are now often used for working loads of 400,000 lb. Mr. Hiley's formula can be adapted arbitrarily to give results which approximately agree with the known facts for these particular piles, but the results of applying the wave equation to the same problem are shown in *Table* 1.

The forces given in *Table* 1 exclude the prestress. The stress in the concrete is therefore greater than that which would be obtained by dividing the forces shown by the cross-sectional area of the pile.

It will be noted that the wave equation shows that only 5 blows per inch are required to drive the pile against an ultimate resistance of 600,000 lb., and that 12·5 blows per inch are sufficient to drive it against 900,000 lb. Refusal does not occur until the resistance reaches 1,200,000 lb. These are in good agreement with the results of tests.

Consideration of *Table* 1 leads to two facts that partly explain the differences obtained by means of the wave-equation calculations and Mr. Hiley's formula.

(1) Up to the instant of maximum penetration of the point the maximum compressive force at the head of the pile (702,000 lb.) is the same for all three calculations even though the resistance at the point varies widely. This is because the pile is heavy enough to stop the ram, with consequent restitution of the dolly and packing, before the reflected stress-wave can return to the head.

(2) In Mr. Hiley's formula the assumption is made that a uniform compressive force equal to the ultimate driving resistance exists throughout the length of the pile at the instant of maximum penetration of the point. The wave-equation calculation shows that this is nearly true for light piles, because the pressure of the ram on the head of the pile is maintained up to and including the instant of maximum penetration of the point, but for long heavy piles the compressive force is not uniform. The force produced at the point may be almost twice as great as the force that was produced by the hammer at the head of the pile. This is due to the almost complete reflection of the compressive stress-wave by the ground. The wave turns back on itself at the point of the pile and thereby momentarily increases its force. The active pressure of the ram on the head has stopped; therefore this increase of force is only momentary, but it effectively overcomes the resistance of the ground unless the pile has reached refusal. At the time Mr. Hiley's formula was developed there was little or no experience of long and heavy piles at all equivalent to the example in *Table* 1, and the possible importance of these factors could not be anticipated.

Fig. 1 shows the kind of results obtainable for shorter piles by wave-equation calculations. The data required for the calculations are given in the following:

Length of pile, 50 ft.; material of pile, steel; weight per foot, as given on the graph. Weight of ram, 5000 lb.; gravity fall, 3 ft.; rated energy per blow, 15,000 ft.-lb.; assumed efficiency, 88 per cent.

Weight of helmet, 400 lb. for pile weighing 25 lb. per foot; 1000 lb. for pile weighing 100 lb. per foot; 1200 lb. for pile weighing 400 lb. per foot (others in proportion).

Quantities that are known or assumed are the elastic modulus of the dolly, 4,000,000 lb. per inch of compression; coefficient of restitution of dolly, 50 per cent.; elastic modulus of pile, 29,300,000 lb. per square inch; ultimate driving resistance, 400,000 lb.; quake of ground or elastic compression, 0·1 in.

In general, it may be concluded from the wave-equation calculations that light piles are driven faster than heavy piles so long as the resistance of the ground is comparatively small, as when driving through the upper ground, but when the resistance begins to increase comparatively heavy piles can best be driven to a firm bearing, and thus can carry more load.

assumed to act at the point. From this it may be tentatively concluded that the assumption that all the resistance occurs at the point gives conservative results when using the wave-equation calculations; this is true also with Mr. Hiley's formula.

Like Mr. Hiley's formula, the wave-equation calculation gives the ultimate driving resistance at the instant of

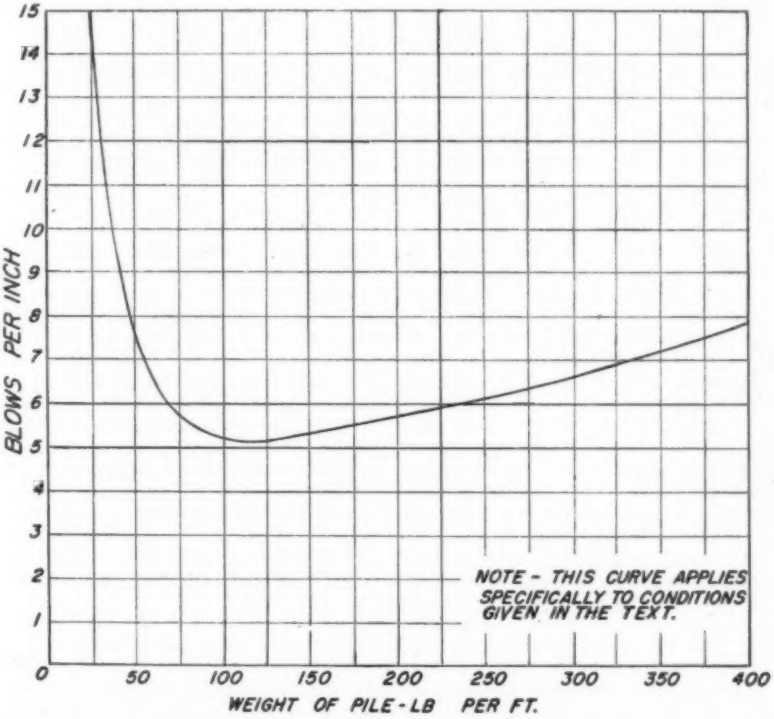


Fig. 1.—Influence of Weight per Foot of Pile on Effectiveness of Hammer Blow.

A few wave-equation calculations have been made assuming that part or all of the ultimate driving resistance acts as side friction along the length of the pile. Various distributions were tried; in some cases the friction was considered to be uniform for the full length and in others a triangular distribution was assumed. In nearly every case it was found that the minimum penetration per blow was obtained when all the resistance was

driving. The relationship between this resistance and the safe load-carrying capacity of the pile is a problem that cannot be discussed at length here. In most cases the wave equation with a factor of safety of 3 or 4 will indicate a permissible load as high or higher than would Mr. Hiley's formula with a factor of safety of $2\frac{1}{2}$.

In conclusion it may be remarked that the attempt to apply wave-equation

calculations to pile-driving has extended over many years. Now that electronic computers can do the difficult mathematical work involved, the project has become practicable.

(1) An Investigation of the Stresses in Reinforced Concrete Piles during Driving. W. H. Glanville, G. Grime, E. N. Fox, and W. W. Davies. Building Research Technical Paper No. 20, D.S.I.R., 1938.

(2) "Impact and Longitudinal Wave Transmission." E. A. Smith. Trans. Am.S.M.E., August 1955.

(3) "What Happens when Hammer hits Pile." E. A. Smith. Engineering News-Record, Sept. 5, 1957.

Reprints of (2) and (3) may be obtained from Raymond International Inc., 140 Cedar Street, New York 6, U.S.A., of which Company the writer is Chief Mechanical Engineer.

Book Reviews.

"The Structures of Eduardo Torroja." (London: Interscience Publishers, Ltd. Price £3 3s.)

THOSE who heard Professor Torroja lecture in London in January last had no doubt that they were listening to a great man, a man to whom mathematics were intuitive, to whom ingenuity came naturally, and a great artist and visionary, who fortunately dedicated his life to architecture and construction to the benefit of the world and to structural engineering in particular. This book sets out to record his more important and interesting achievements. It does so magnificently. The structures are illustrated with photographs and drawings showing clearly the new conceptions in structural shapes with which he has enriched his native Spain. But the work is much more than a picture book, for Professor Torroja not only describes the design of the structures as they now stand but he also discusses the reasoning that led up to them. In some cases his earlier thoughts are also explained and illustrated, and the reasons are given why they were discarded. The philosophy of his approach to structural problems is clearly presented, and we are able to understand the mental processes that have resulted in shapes and designs not hitherto attempted in concrete. A stimulating book that should be read and reread by structural engineers—except perhaps those who are unwilling or unable to dream, and who believe that their work comprises no more than following existing types of structures and designing according to codes and handbooks. And men who believe that a smattering of mathematics is the chief qualification of an engineer will be surprised to know that such a brilliant mathematician sets more store on the results of the testing of scale models.

"Materials of Civil Engineering." By J. P. M. Pannell. (London: Hutchinson & Co., Ltd. 248 pages. Price 30s.)

THIS book provides a useful and accurate introduction to the properties and uses of soils and rocks, timber, iron, steel, aluminium, cement and concrete, and chemical products. A great deal of information is given, but the treatment is uneven; it is surprising to see one-fifth of the book devoted to timber and only eleven pages to paint, rubber, plastics, bitumen, and asphalt. The description of this group of products is superficial and could usefully have been enlarged at the expense of other sections. A more detailed and extensive bibliography would also have been of value.

"Praktische Winke zum Studium der Statik." By Ernst Kohl. (Berlin: Springer-Verlag. Price 18 D.M.)

IN this short work on the theory of structures the author deals with problems of equilibrium, influence lines, the solution of deformation problems, transversely-loaded beams subjected to longitudinal forces, selected statically-indeterminate structures, and the stability of frames. In the section on statically-indeterminate structures, strictly accurate proofs are frequently abandoned in favour of geometrical and graphical solutions. There are numerous exercises, and the value of these is enhanced by giving possible alternative solutions with a comparison of the advantages and disadvantages in each case. The book is primarily intended for students, but practising engineers may find it useful for refreshing their knowledge of the principles of statics.

"A.C.I. Manual of Concrete Inspection" (price $3.50); **"A.C.I. Standards, 1957"** (price $4); **"Manual of Standard Practice for Detailing Reinforced Concrete Structures"** (price $4). (Detroit: American Concrete Institute.)

THE 1957 edition of the Manual of Concrete Inspection maintains the high standard of the previous editions. All aspects of the subject are comprehensively and concisely described, and the book is useful to anyone concerned with the production and inspection of concrete.

The book of A.C.I. Standards includes eleven current U.S.A. Standards concerning most aspects of the design and manufacture of concrete. The Manual of Standard Practice is of little assistance to British engineers, in view of the wide differences between British and U.S. practice.

"Der Stahlbetonbau." By Karl Kersten. Revised by Heinrich Kuhnert. Part III. Entwurfsbeispiele im Hoch und Industriebau. (Berlin: Wilhelm Ernst & Son. 1957. Price D.M. 23.50.)

THIS edition includes 120 examples of reinforced concrete design applied to buildings, including slabs and beams, columns and footings, staircases, roofs, frames, and free-standing walls. Continuous beams and rectangular frames are designed by the theory of three moments or the method of moment-distribution. The designs are based upon the German draft code of practice DIN 1040, which does not require the determination of exact values of the stresses in concrete provided that they do not exceed the maximum permissible values. By the use of two or three tables (no diagrams are needed) it is possible to calculate the whole range of bending combined with axial forces (of which bending only is treated as a particular case), resulting in an appreciable saving of time.

British Standard Code of Practice No. 2001 (1957): Site Investigations. (London: British Standards Institution. Price 20*s*.)

THIS is a revised edition of the Civil Engineering Code of Practice No. 1 which was published by the Institution of Civil Engineers in 1950. It deals mainly with the investigation of the suitability and characteristics of sites as they affect the design and construction of civil engineering works.

"Earth Pressures and Retaining Walls." By Whitney Clark Huntingdon. (London: Chapman & Hall, Ltd. Price 92*s*.)

THIS book of 534 pages is devoted entirely to the design of retaining walls and the forces to which they are subjected. It is a comprehensive and thorough treatment of every aspect of the subject; both text and diagrams are clear, and the examples are well chosen. The methods described are widely used in this country as well as in the U.S.A., and engineers confronted with unusual problems relating to retaining walls will almost certainly find the solutions here.

Best Quality

CONCRETE AGGREGATES

in all grades

DELIVERED BY ROAD OR RAIL

WM. BOYER & SONS LTD

MEMBERS OF S. & G.A. of G.B.

ESTABLISHED 1810

IRONGATE WHARF, PADDINGTON, W2

TELEPHONE: PADDINGTON 2024-6

A Prestressed Bridge at Nottingham.

THE new Clifton bridge (*Figs.* 1 and 2) over the River Trent at Nottingham has an overall width of 46 ft. 3 in., including a 22-ft. carriageway, a 9-ft. cycle track, and a 6-ft. footpath. The angle of skew to the river is 24 deg.

The main bridge consists of a central span of 275 ft. over the normal bed of the river (this is the longest prestressed concrete span in Great Britain) and two side spans of 125 ft. (*Figs.* 3 and 4). On the north the approach road is partly on an embankment and partly on a flood-relief viaduct of three 40-ft. spans. On the south a viaduct of three 90-ft. spans adjoins the bridge; the total continuous length, excluding the northern viaduct, is almost 800 ft.

The bridge is designed in accordance with the Ministry of Transport standards for abnormal loading. The Magnel-Blaton system of prestressing was used for the main bridge and the southern viaducts; the northern viaducts are prestressed by the Lee-McCall system. Detailed studies were conducted by the consulting engineers on a perspex model of one-fiftieth size to determine the effect of the skew.

The Main Bridge.

The main bridge is designed as two cantilevers each 87 ft. 6 in. long supporting a suspended central span 100 ft. long, and balanced by end spans of 125 ft. The end spans and cantilevers were cast in

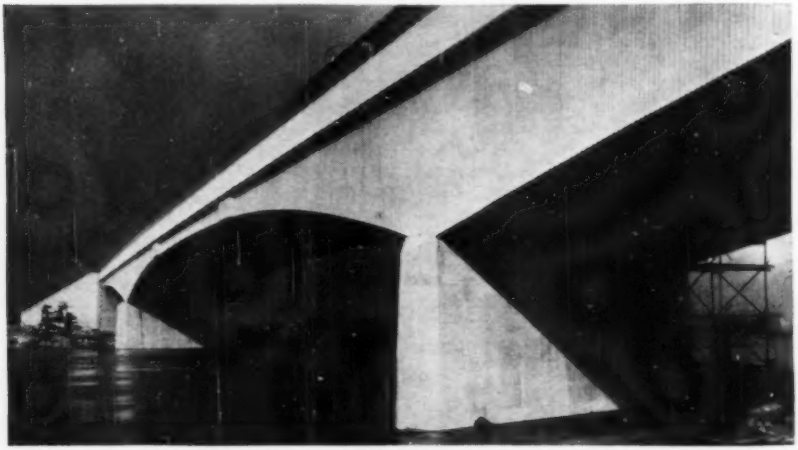


Fig. 1.

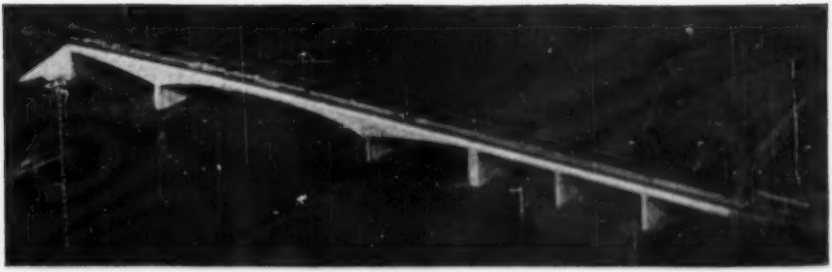


Fig. 2.

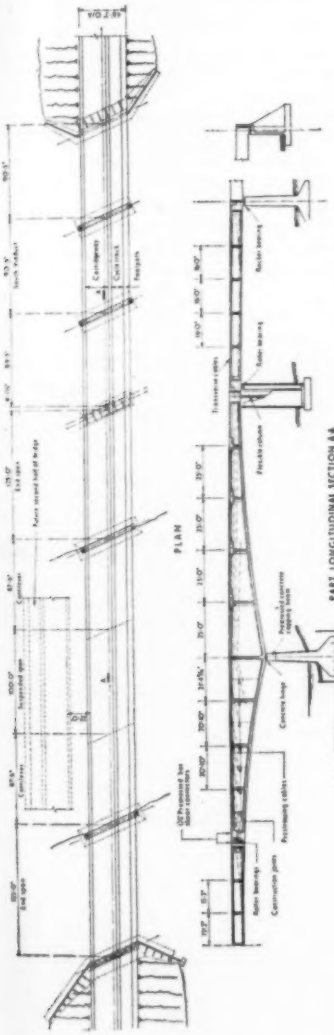


Fig. 3.—Part Plan and Longitudinal Section.

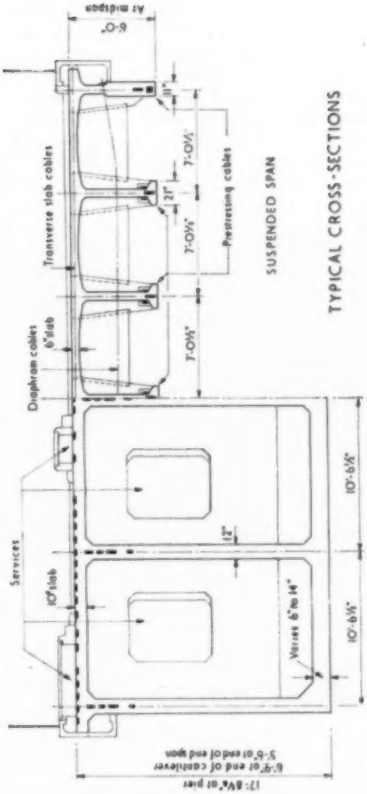


Fig. 4.—Transverse Sections.

place; precast beams are used in the central span.

The abutment at the north end is in the form of a cellular retaining-wall, with wing walls of counterfort construction surrounding the end of the embankment. At the south end a cellular abutment supports the end span of the main bridge and the first span of the viaduct. Flexible prestressed columns within the cellular portion of each abutment support the end span. Two solid piers of reinforced concrete support the cantilevers and the end spans at the river banks; they were built on concrete piles within a cofferdam. A prestressed capping beam surmounts the piers; there is a concrete hinge between this and the bridge beams.

The balanced cantilevers are of cellular construction with connecting webs and stiffeners 12 in. thick. The longitudinal webs are about 10 ft. apart; the transverse diaphragms are at intervals of 25 ft. in the end spans and about 21 ft. in the cantilevers. The 10-in. top slab forms the deck; the bottom slab varies in thickness from 14 in. at the pier to 6 in. at the ends of the span. Heavy service mains pass through the stiffeners and under the precast beams of the central span; lighter services are accommodated under the precast footpath and the precast verge between the cycle-path and the carriageway.

The end spans were built on falsework in the usual manner, and the cantilevers were then constructed in 10-ft. sections with the aid of a moving scaffold suspended from the previously-completed section. The prestressing was so designed that as each section of the cantilever was completed enough cables were anchored at the end of the section to provide sufficient strength for the construction of the next stage. The prestressing wires are in the deck slab and the longitudinal webs.

The suspended span consists of five I-beams and two T-beams, 6 ft. 9 in. deep at the ends and 6 ft. deep at mid-span, which were precast on the site and partly prestressed. The beams are supported at about 7 ft. centres on roller-bearings at one end and rockers at the other; shear connectors consisting of 1¼-in. high-tensile bars were installed in the concrete at these points and at the ends of the cantilevers. After erection the stiffeners were cast and partly prestressed; the deck slab was then cast and transversely prestressed at the same time as the full prestress was applied to the stiffeners.

The Viaducts.

Each span of the northern viaduct consists of sixteen precast I-beams 2 ft. 6 in. deep, prestressed before erection by the Lee-McCall system. When the beams were in position precast planks were laid between the top flanges to provide permanent shuttering for the deck slab. The stiffeners between the beams were precast and prestressed after the deck was cast. The supports consist of two abutments in the form of counterfort retaining walls and two hinged piers.

Each span of the southern viaduct consists of five central I-beams and two hollow beams; all are 3 ft. 6 in. deep, and prestressed by the Magnel-Blaton system. The central span of the viaduct was cast in place. The beams for the other two spans were precast and prestressed on this completed span before being launched into position. The deck slab and stiffeners were cast in place and transversely prestressed after the concrete had matured. The system of beams and stiffeners was designed as a grillage. The supports consist of solid fixed piers and an abutment in the form of a counterfort retaining-wall with wing walls.

Mr. R. M. Finch, O.B.E., is the City Engineer and Surveyor, the consulting engineers for the main bridge were Messrs. R. Travers Morgan & Partners, and the contractors were Messrs. James Miller & Partners, Ltd.

Lectures on Road Materials.

COURSES of lectures on the properties of road materials and the application of the results of research will be held at the Road Research Laboratory during the latter part of 1958. Details can be had from the Director, Road Research Laboratory, Harmondsworth, West Drayton, Middlesex.

CONCRETE MIX
HOW MUCH WATER?

"Speedy" Moisture Tester gives accurate answer in 3 minutes. Also for moisture content of soil in road construction. Labourers can use on site; no skill; no electricity; portable. Complete, £38. Send for brochure and D.S.I.R. Report to (Dept. C.C.3.) THOS. ASHWORTH & CO. LTD., Vulcan Works, Burnley, Lancs.

Extending Precast Prestressed Piles.

A PLASTIC JOINTING MATERIAL

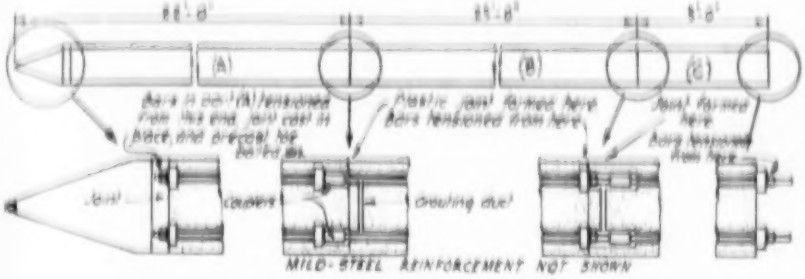


Fig. 1.—Details of Pile.

THE use of an isothermic polyester resin as a jointing material was demonstrated recently during a test on a precast prestressed pile. The material is supplied in the form of a resinous liquid and a powder, which are mixed together to form the jointing material.

The pile tested is shown in *Fig.* 1. The bottom part (A) was first driven until it projected about 4 ft. above the ground, the extension (B) was lifted and aligned, and prestressing bars in the extension were lowered into the open ends of couplers provided in the head of the driven portion. The jointing material was placed on the top of part (A) and the extension was then lowered into position. The material takes about fifteen minutes to set, and during this period preparations were made for the prestressing operation.

After the jointing material had set, a prestressing force of 80 tons was applied to the extension and the pile was driven. The procedure was repeated with extension (C), and driving was continued until the

Fig. 2.—Pile head before forming the Joint

Fig. 3.—Joint before driving.

Fig. 4.—Joint after driving.

set was ¼ in. per blow using a drop-hammer weighing four tons, with no sign of extrusion or distress at the joints. Extension (C) was later pulled off; the joint remained unbroken, failure occurring in the concrete immediately above it. *Fig.* 2 shows the head of part (A) before the joint was formed; *Figs.* 3 and 4 show the joint before and after driving.

The jointing material had a compressive strength of about 12,000 lb. per square inch and a tensile strength of about 1500 lb. per square inch. It is stated that by varying the grade of powder compressive strengths up to 20,000 lb. per square inch and tensile strengths up to 10,000 lb. per square inch can be obtained. The material can be used under water, in which case its properties are slightly reduced. It is stated to have a shrinkage of about 2 per cent., a low elastic modulus, and to be resistant to chemical attack. Similar materials can be applied by means of a double-nozzle spray gun to form a wearing surface on slabs or floors.

The material, known as "Artrite 13-10", is supplied by Artrite Resins, Ltd. The pile tested was driven by the Demolition & Construction Co., Ltd., as part of a contract for the South Eastern Gas Board at Wandsworth, London. The test was made on behalf of McCalls Macalloy, Ltd., with the collaboration of the Board.

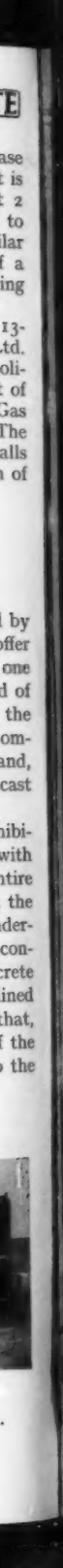
FIFTY YEARS AGO.

From "Concrete and Constructional Engineering," May–June, 1908.*

Reinforced Concrete Framed Building.—The office buildings recently erected by Messrs. Holland & Hannen, of London, known as Friars House, Broad Street, offer an interesting example of modern methods of construction. It is, we believe, one of the first modern buildings in the City of London to be erected without the aid of structural steel of any kind; in fact, there is not a single rolled steel joist in the building. . . . The concrete was hand-mixed, twice dry and twice wet, and composed of one part "Ferrocrete" Portland cement, two parts washed Thames sand, and three parts Thames ballast, crushed to pass a ¾ in. mesh. The mixture was cast moderately wet and gave a most satisfactory result.

The Franco-British Exhibition.—The Stadium at the Franco-British Exhibition [in London] is an excellent instance of how to employ a framework of steel with platforms of reinforced concrete. It was originally proposed to erect the entire structure in reinforced concrete but apparently the question of demolishing it at the termination of the Exhibition caused the decision in favour of steel. We understand that there was practically no difference in the cost of the two forms of construction, though, if anything, reinforced concrete was the cheaper. . . . The concrete for the reinforced concrete platform was placed in position by means of an inclined tramway running up the Stadium. The truck consisted of four platforms so that, when stopped, each platform was opposite in line and level with the platform of the Stadium itself. The workmen were thus able to wheel the barrows direct on to the work. The centering for the concrete was all slung.

* "Concrete and Constructional Engineering" appeared in alternate months until September, 1909.

A Portable Cement Silo.

The portable cement silo shown in the illustration is of plywood on a steel frame, and can be towed by a 1-ton lorry. It has a capacity of up to 40 tons of cement, and is fitted with a 7½-h.p. electric motor for operating a rotary blower for discharging the contents. A beam-type scale, which can be operated automatically or by hand, is provided. The silo is supplied by Road Machines (Drayton), Ltd.